Non-Perturbative Flat Direction Decay

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We argue that supersymmetric flat direction vevs can decay non-perturbatively via preheating. Considering the case of a single flat direction, we explicitly calculate the scalar potential in the unitary gauge for a $U(1)$ theory and show that the mass matrix for excitations around the flat direction has non-diagonal entries which vary with the phase of the flat direction vev. Furthermore, this mass matrix has 2 zero eigenvalues (associated with the excitations along the flat direction) whose eigenstates change with time. We show that these 2 light degrees of freedom are produced copiously in the non-perturbative decay of the flat direction vev. We also comment on the application of these results to the MSSM flat direction H_uL.

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INTRODUCTION

The scalar potential of the Minimal Supersymmetric Standard Model (MSSM) possesses a large number of F- and D-flat directions along which the scalar potential nearly vanishes [1, 2]. These flat directions can have important cosmological consequences, including the generation of the baryon asymmetry of the Universe through the out-of-equilibrium CP violating decay of coherent field oscillations along the flat directions themselves [3, 4, 5].

Recently, much interest has focused on the cosmological fate of flat direction vevs. In particular, it has been argued [6] that in realistic supersymmetric models, large flat direction vevs can persist long enough to delay thermalization after inflation and therefore lead to low reheat temperatures. Furthermore, it has also been asserted [7] that large flat direction vevs can prevent non-perturbative parametric resonant decay (preheating) of the inflaton since the inflaton decay products become sufficiently massive and thus prevent preheating from ever becoming efficient. The above arguments hold so long as the flat direction vevs do not rapidly decay – they must persist long enough so that they can delay thermalization and block inflaton preheating. In [8] it was claimed that non-perturbative decay can lead to a rapid depletion of the flat direction condensate and thus precludes the delay of thermalization after inflation. It was also concluded that in order for the flat direction to decay non-perturbatively the system requires more than one flat direction with some degree of fine tuning [8, 9].

The most germane aspect of this discussion centers on the issue of Nambu-Goldstone (NG) bosons. In general, supersymmetric flat directions are charged under the gauge group of the MSSM. Consequently, the flat direction vev will break some or all of the gauge symmetries of the theory and thus we expect the presence of associated NG bosons. In calculating non-perturbative flat direction decays, [8] considers a gauged $U(1)$ model and constructs the mixing matrix for the excitations around the flat direction vev. The results in [8] show that in the single flat direction case, non-perturbative decay proceeds solely via a massless NG mode as only the NG mode mixes with the Higgs and all other massless moduli remain decoupled. Since the NG boson represents an unphysical gauge degree of freedom, it was concluded [8, 9] that no preheating occurs in the single flat direction case. As the appearance of a massless NG boson in the spectrum is a gauge dependent artifact, it remains unclear if the conclusions drawn about the system holds in the unitary gauge. In order to determine if non-perturbative flat direction vevs decay into scalar degrees of freedom, the effect of the NG boson mixing in the scalar potential must first be removed. The process of removing the NG modes by switching to the unitary gauge changes the form of the mixing matrix among the left over scalar degrees of freedom. The resulting form of the mass matrix in principle can permit non-perturbative decay.

In this letter we demonstrate that, in the unitary gauge, the mixing matrix of the excitations around the flat direction vev permits preheating with a single flat direction. In particular, the moduli of the flat direction always remain light (as compared to the flat direction vev) and will in principle possess time dependent eigenstates, thereby satisfying the necessary (but not sufficient) condition for preheating. The outline of the rest of this letter proceeds as follows: firstly we explicitly construct – in the unitary gauge – the mass squared matrix arising from the D-terms of a toy gauged $U(1)$ model with two charged chiral superfields. We then present the formalism of preheating with multi-component fields and show that preheating occurs for the light moduli associated with the flat direction. We also comment on the applications to a realistic flat direction, namely H_uL. Lastly, we conclude with a summary of our findings.

TOY MODEL: GAUGED $U(1)$

Firstly, we wish to examine a toy model which demonstrates the most important features of single supersymmetric flat direction vev decay. Following the example of [8], we introduce two complex scalar superfields, Φ_1 and Φ_2 charged under a $U(1)$ gauge group with charges $+1/2$ and $-1/2$ respectively. The potential we consider arises from the supersymmetric D-terms and has the form,

$$V = \frac{g^2}{8}(|\Phi_1|^2 - |\Phi_2|^2)^2, \tag{1}$$

where g denotes the gauge coupling associated with the $U(1)$ gauge symmetry. We have neglected all contributions from supersymmetry breaking and from any terms arising from the superpotential. These terms are only significant in the computation of the evolution of the flat direction vev, see [10, 11].

The potential in eq.1 admits a flat direction defined by,

$$\langle \Phi_1 \Phi_2 \rangle = \varphi^2 e^{i\sigma}, \tag{2}$$

where φ and σ are real and time dependent. Note that in this example the two chiral superfields contain two complex scalar degrees of freedom. The flat direction vev breaks the $U(1)$ symmetry yielding one massive Higgs field and one NG boson, leaving two massless scalar degrees of freedom.

In order to find the mass matrix with the physical degrees of freedom, we expand the excitations of the fields around their background values in the unitary gauge à la Kibble [12]. An appropriate form which still obeys D-flatness is given by,

$$\begin{aligned} \Phi_1 &= \varphi + \xi_1 \\ \Phi_2 &= \varphi e^{i\sigma} + \xi_2 + i\xi_3. \end{aligned} \tag{3}$$

The scalar potential now appears with three physical scalar degrees of freedom with the NG boson associated with the broken $U(1)$ removed. Substituting these forms into the potential in eq.1, we find the quadratic form

$$V \supset \frac{(g\varphi)^2}{2}(\xi_1 - \xi_2\cos\sigma - \xi_3\sin\sigma)^2, \tag{4}$$

leading to the mass squared matrix

$$V \supset \frac{1}{2}\begin{pmatrix} \xi_1 & \xi_2 & \xi_3 \end{pmatrix} \mathcal{M}^2 \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix} \tag{5}$$

with

$$\mathcal{M}^2 = (g\varphi)^2 \begin{pmatrix} 1 & -\cos\sigma & -\sin\sigma \\ -\cos\sigma & \cos^2\sigma & \cos\sigma\sin\sigma \\ -\sin\sigma & \cos\sigma\sin\sigma & \sin^2\sigma \end{pmatrix}. \tag{6}$$

The eigenvalues of eq.6 are $M_1^2 = 2(g\varphi)^2$, $M_2^2 = 0$, $M_3^2 = 0$. M_1 corresponds to the mass of the physical Higgs particle associated with the spontaneous breaking of the $U(1)$ symmetry, while the two zero eigenvalue states correspond to the excitations around the flat direction vev parameterizing the flat direction moduli space.

The corresponding eigenstates read,

$$\begin{aligned} \hat{\xi}_1 &= \frac{1}{\sqrt{2}}\left(-1,\ \cos\sigma,\ \sin\sigma \right), \\ \hat{\xi}_2 &= \left(0,\ \sin\sigma,\ -\cos\sigma \right), \\ \hat{\xi}_3 &= \frac{1}{\sqrt{2}}\left(1,\ \cos\sigma,\ \sin\sigma \right), \end{aligned} \tag{7}$$

where $\hat{\xi}_i$ has mass M_i.

We should note at this point that if we include contributions arising from the kinetic terms, we will generate extra mass terms which mix the scalar fields with the massive gauge boson. In the limit where $\dot{\sigma} \ll \varphi$ and $\dot{\varphi} \ll \varphi^2$, these mass terms are negligible compared to the large mass of the gauge boson (which is proportional to φ) and consequently the gauge boson decouples from the scalar sector and can be ignored in this analysis.

The central point of this discussion is the appearance of *time dependent* eigenvectors for the 2 remaining light fields. Furthermore, these light states are produced during preheating. In the next section, we demonstrate the effect of preheating following the analysis in [13].

NON-PERTURBATIVE PRODUCTION OF PARTICLES

Taking gravity into account, the dynamics of the rescaled conformally coupled scalar fields, $\chi_i = a\xi_i$, where a denotes the scale factor, is governed by the following equations of motion (sum over repeated indices is implied)

$$\ddot{\chi}_i + \Omega_{ij}^2(t)\chi_j = 0 \tag{8}$$

where t is conformal time and

$$\Omega_{ij}^2 = a^2\mathcal{M}_{ij}^2 + k^2\delta_{ij}, \tag{9}$$

where k labels the comoving momentum. Using an orthogonal time-dependent matrix $C(t)$, we can diagonalize Ω_{ij} via $C^T(t)\Omega^2(t)C(t) = \omega^2(t)$, giving the diagonal entries $\omega_j^2(t)$. Terms such as $\sim \varphi\dot{\sigma}\dot{\chi}$ arising from the kinetic part of the Lagrangian do not affect the evolution of the non-zero k quantum modes [14].

Once we have identified the basis in which the Hamiltonian appears diagonal (via the orthogonal matrix $C(t)$), the study of particle creation by the time-varying background proceeds as [13, 15], which extends the results of [16]. Following [13], we assume that Ω_{ij} initially evolves adiabatically which we can do if we assume the initial angular motion of the flat-direction is very slow allowing us to define adiabatically evolving mode functions with positive and negative frequency. We rewrite the quantum fields as mode expansions in terms of these mode functions and their associated creation/annihilation operators which allows us to define the initial vacuum. During the evolution, the entries of Ω_{ij} are no longer assumed to change adiabatically and consequently we must find new mode functions that satisfy eq.8. A new set of creation/annihilation operators required to define the new vacuum can be related to the initial set using a Bogolyubov transformation with Bogolyubov coefficients α and β which are matrices in the multi-field case.

Initially $\alpha = \mathbb{I}$ and $\beta = 0$ while the coupled differential equations (matrix multiplication is implied):

$$\begin{aligned} \dot{\alpha} &= -i\omega\alpha + \frac{\dot{\omega}}{2\omega}\beta - I\alpha - J\beta \\ \dot{\beta} &= \frac{\dot{\omega}}{2\omega}\alpha + i\omega\beta - J\alpha - I\beta \end{aligned} \tag{10}$$

govern the system's time evolution with the matrices I and J given by,

$$I = \frac{1}{2}\left(\sqrt{\omega}\,C^T\dot{C}\frac{1}{\sqrt{\omega}} + \frac{1}{\sqrt{\omega}}C^T\dot{C}\sqrt{\omega}\right); \quad (11)$$

$$J = \frac{1}{2}\left(\sqrt{\omega}\,C^T\dot{C}\frac{1}{\sqrt{\omega}} - \frac{1}{\sqrt{\omega}}C^T\dot{C}\sqrt{\omega}\right). \quad (12)$$

Dots represent derivatives with respect to conformal time. Similarly to the single-field case it can be shown [13] that at any generic time the occupation number of the ith bosonic eigenstate reads,

$$n_i(t) = (\beta^*\beta^T)_{ii}. \quad (13)$$

As pointed out in [8, 13], there are two sources of non-adiabaticity in the multi-field scenario. The first source arises from the individual frequency time dependence and appears as the only source of non-adiabaticity in the single field case. The second source appears from the time dependence of the frequency matrix Ω_{ij} giving rise to terms in eq.10 proportional to I and J. This second source provides the most important contribution in our analysis and gives rise to non-perturbative particle production.

NUMERICAL ANALYSIS

We use the potential [8],

$$V = \frac{1}{2}m_\phi^2|\phi|^2 + \frac{\lambda}{8}(\phi^4 + \text{h.c.}), \quad (14)$$

to calculate the classical evolution of the flat direction vev, ϕ, and analyze particle production in a self-consistent background for the $U(1)$ model described above. In eq.14, m_ϕ denotes a supersymmetry breaking mass term ($\sim$ TeV) and λ (arising from loop effects) is expected to be on the order of $\sim g^4 m_\phi^2/\varphi_0^2$ [3] (φ_0 denotes the time-independent initial value of the flat direction vev). Measuring the conformal time in units of $\tau \to f\tau$ with $f = g\varphi_0$ and using the rescaled flat-direction vev

$$\phi = \frac{\varphi_0}{a}Fe^{i\sigma}$$

we find the background equations

$$F'' + \left[\frac{\mu^2 a^2}{2} - \sigma'^2 - \frac{a''}{a}\right]F + \frac{\lambda}{2g^2}F^3\cos(4\sigma) = 0, \quad (15)$$

where a prime represent derivative with respect to τ and

$$\sigma'' + 2\sigma'\frac{F'}{F} - \frac{\lambda}{2g^2}F^2\sin(4\sigma) = 0, \quad (16)$$

which describe the motion of the flat direction vev; $\mu = m_\phi/f$. The scale factor evolves as

$$\frac{a''}{a} = -\frac{a'^2}{a^2} + \frac{1}{2}\left[f_p^2\left\{\mu^2F^2 + \frac{\lambda}{2g^2}\frac{F^4}{a^2}\cos(4\sigma)\right\} + \frac{a^2\rho_\psi}{M_{pl}^2f^2}\right]$$

where ρ_ψ is the energy density of the inflaton field, and $f_p = \varphi_0/M_{pl}$ is set to $f_p = 0.1$ in our numerics. We also take $\mu = 10^{-2}$ and $\lambda = \mu^2$ for computational ease. As initial conditions we take $F = 1$, $\sigma = 0.1$, $\sigma' = 0$, $a = 1$ and $\phi' = 0$. Using the rescaled Hubble parameter $H \equiv fh$ as in [8], we define the initial conditions of F' and a' through $h_i = \mathcal{F}\mu$ where h_i denotes the Hubble parameter after inflation. We take $\mathcal{F} = 10^2$ for illustrative purposes.

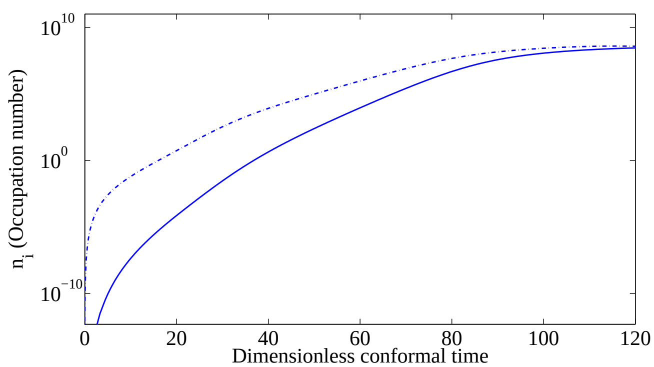

FIG. 1: Occupation numbers as a function of dimensionless conformal time, obtained using eq.13 after numerically integrating the background field equations and Bogolyubov matrices; $k = 10^{-6}, \mu = 10^{-2}, \mathcal{F} = 10^2$. The solid line and the dashed-dotted line distinguish the two massless modes.

Initially the flat direction vev corresponds to a condensate of coherent particles with vanishing momentum. The motion of this vev, described by eq.15 and eq.16, and the interactions described in the previous section, cause the rapid decay of this condensate into a decoherent state of particles. FIG. 1 shows the occupation numbers, $n_i(t)$, of these light particles as a function of conformal time: the exponential growth of these functions signals the exponentially fast decay of the flat-direction vev. The resulting decoherent spectrum of light particles is shown in FIG. 2 as a function of the comoving momentum. Production of higher momentum modes is kinematically suppressed.

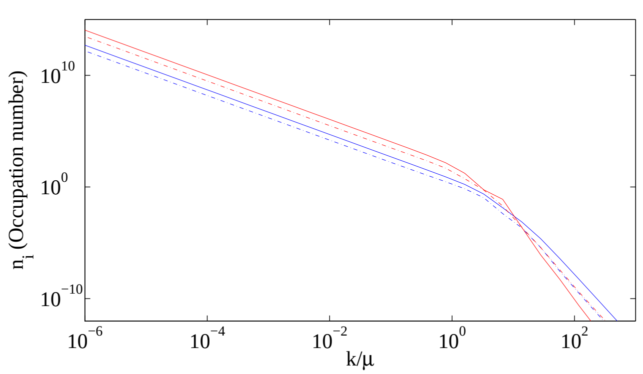

FIG. 2: Occupation number as function of the (scaled) comoving momentum; $\mu = 10^{-2}, \mathcal{F} = 10^2$ for dimensionless conformal time $\tau = 150$(blue), 300(red). The solid line and the dashed-dotted line distinguish the two massless modes.

REALISTIC EXAMPLE: H_uL

The flat direction H_uL presents an interesting example as it carries net $B-L$, thus its decay can lead to Affleck-Dine leptogenesis [3, 5, 17]. Ignoring contributions from supersymmetry breaking and the superpotential, the scalar potential arising from D-terms appears as,

$$V = \frac{g_1^2}{2}(|H_{ui}|^2 - |L_i|^2)^2 + \frac{g_2^2}{2}(H_{ui}^* \tau_{ij}^a H_{uj} + L_i^* \tau_{ij}^a L_j)^2$$

where g_1 and g_2 are the gauge couplings associated with the $U(1)_Y$ and $SU(2)_L$ gauge symmetries respectively, and $\tau^a = \sigma^a/2$ where σ^a denote the Pauli matrices. The index a is summed over the three generators of $SU(2)_L$. The Roman numeral indices label the components of each $SU(2)$ doublet and again are summed over. The H_uL flat direction can be defined as

$$\langle H_u L \rangle = \varphi^2 e^{i\sigma}. \tag{17}$$

The chiral superfields contain four complex scalar fields and since the symmetry breaking appears as $SU(2)_L \times U(1)_Y \to U(1)_{\rm em}$, the resulting degrees of freedom are: three NG bosons, three massive Higgs fields, and two massless scalar fields.

We can expand the fields about their background values following Kibble [12] and, after imposing D-flatness and eq.17, we can write the physical excitations in the unitary gauge as

$$H_u = \begin{pmatrix} 0 \\ \varphi + \xi_1 \end{pmatrix} \quad L = \begin{pmatrix} \varphi e^{i\sigma} + \xi_2 + i\xi_3 \\ \xi_4 + i\xi_5 \end{pmatrix}. \tag{18}$$

The resulting mass terms appear as,

$$V \supset \frac{1}{2} \begin{pmatrix} \xi_1 & \xi_2 & \xi_3 \end{pmatrix} \mathcal{M} \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix} + \frac{\varphi^2}{2} g_2^2 (\xi_4^2 + \xi_5^2) \tag{19}$$

where,

$$\mathcal{M} = \varphi^2 (g_1^2 + g_2^2) \begin{pmatrix} 1 & -\cos\sigma & -\sin\sigma \\ -\cos\sigma & \cos^2\sigma & \cos\sigma\sin\sigma \\ -\sin\sigma & \cos\sigma\sin\sigma & \sin^2\sigma \end{pmatrix}.$$

In this case, the mass spectra reads : $M_1^2 = 2\varphi^2(g_1^2 + g_2^2)$, $M_2^2 = 0$, $M_3^2 = 0$, $M_4^2 = \varphi^2 g_2^2$, $M_5^2 = \varphi^2 g_2^2$. Comparison with eq.6 shows that the mixing in this case appears exactly as the gauged $U(1)$ example considered above with the sole addition of two unmixed massive Higgs states. Thus, the flat-direction H_uL undergoes resonant decay.

CONCLUSIONS

The cosmological fate of flat directions provide a major ingredient for the history of the early Universe. Flat directions can provide mechanisms for generating the baryon asymmetry of the Universe and can play an important role in reheating after inflation. Contrary to claims that single supersymmmetric flat direction vevs cannot decay non-perturbatively [8, 9], we have shown explicitly that preheating can occur with a single flat direction. Our analysis stresses the use of the unitary gauge in which the physical content of the theory becomes manifest. By transforming to the unitary gauge, complications arising from massless NG modes in the mixing of the excitations around the flat direction vev vanish. The mixing matrix in this gauge defines the mass eigenstates of the physical scalar fields and determines if non-perturbative decay occurs. Since the mass matrix in the unitary gauge contains time dependent mixing among all fields, the necessary conditions for preheating are satisfied.

We also show that preheating appears in the more realistic flat direction case of H_uL. Moreover, we expect that once the NG modes associated with any supersymmetric flat direction are removed, and a consistent unitary gauge calculation of the mixing matrix is carried out, preheating will appear as a generic feature.

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[1] T. Gherghetta, C. F. Kolda and S. P. Martin, Nucl. Phys. B **468**, 37 (1996).
[2] For a review, see K. Enqvist and A. Mazumdar, Phys. Rept. **380**, 99 (2003).
[3] I. Affleck and M. Dine, Nucl. Phys. B **249**, 361 (1985).
[4] A. D. Linde, Phys. Lett. B **160**, 243 (1985).
[5] M. Dine, L. Randall and S. D. Thomas, Phys. Rev. Lett. **75**, 398 (1995).
[6] See eg., R. Allahverdi, K. Enqvist, J. Garcia-Bellido and A. Mazumdar, Phys. Rev. Lett. **97** (2006) 191304.
[7] R. Allahverdi and A. Mazumdar, [arXiv:hep-ph/0603244].
[8] K. A. Olive and M. Peloso, Phys. Rev. D **74**, 103514 (2006).
[9] R. Allahverdi and A. Mazumdar, [arXiv:hep-ph/0608296].
[10] M. Postma and A. Mazumdar, JCAP **0401**, 005 (2004).
[11] R. Allahverdi and B. A. Campbell, Phys. Lett. B **395**, 169 (1997).
[12] T. W. B. Kibble Phys. Rev. **155** (1967) 1554.
[13] H. P. Nilles, M. Peloso and L. Sorbo, JHEP **0104** (2001) 004.
[14] R. Casadio, P. L. Iafelice and G. P. Vacca, [arXiv:hep-th/0702175].
[15] L. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Rev. D **56** (1997) 3258.
[16] Y. B. Zeldovich and A. A. Starobinsky, Sov. Phys. JETP **34** (1972) 1159 [Zh. Eksp. Teor. Fiz. **61** (1971) 2161].
[17] H. Murayama and T. Yanagida, Phys. Lett. B **322** (1994) 349.